**Unit 4: Fractions & Ratios, Proportions**

**Lecture Notes:**

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| **Main Ideas** | **Details** |
| **Vocabulary** | * **Numerator=top number** * **Denominator=bottom number** * **Proper fraction=numerator smaller than denominator** * **Ex: 6/13** * **Improper fraction=denominator smaller than numerator** * **Ex: 13/6** * **Mixed number=integer and proper fraction; all improper fractions can be converted to mixed numbers** * **Ex: 2 1/6** |
| **How to Convert an Improper Fraction to a Mixed Number** | 1. **Divide the numerator by the denominator. (Do division with remainders, not division with decimals.)** 2. **The quotient is the whole number.** 3. **The remainder is the numerator.** 4. **The divisor is the denominator**  * **Ex: Convert 13/6 to a mixed number** |
| **How to Convert a Mixed Number to an Improper Fraction** | * **“Check mark method”**  1. **Multiply the whole number by the denominator.** 2. **Add the numerator to the product.** 3. **Keep the denominator the same.**  * **Ex: Convert 2 1/6 to an improper fraction** |
| **Equivalent Fractions** | * **You can arrive at an equivalent fraction by multiplying or dividing both the numerator and the denominator by the SAME number** * **Ex: Find three equivalent fractions for 5/10** |
| **Reducing to Lowest Terms** | * **Making the numerator and the denominator the lowest numbers possible by dividing until you reach the smallest number possible**  1. **Find a common factor.** 2. **Divide both the numerator and the denominator by the SAME factor.** 3. **Continue to find common factors until the numerator and denominator have no more factors in common.**  * **Ex: What is 50/100 in lowest terms?** |
| **Greatest Common Factor (GCF)** | * **“Factor Tree” (NOT Prime Factorization)** * **Finding the GCF will assist in quickly reducing the fraction to lowest terms.** * **There will only be one step to reducing the fraction to lowest terms if you divide by the GCF.** * **Ex: 56/80** |
| **Converting Decimals to Fractions** | * **By reading the decimal properly, you “say” the fraction form immediately.** * **Ex: 0.5 is what fraction?** * **(You may have to reduce to lowest terms)** |
| **Converting Fractions to Decimals** | * **The fraction bar functions as a division symbol.** * **Divide the numerator by the denominator to covert a fraction to a decimal.** * **Ex: ¾** |
| **Non-terminating decimals** | * **Some fractions, when converted decimals, result in non-terminating decimals.** * **Ex: 1/3** |
| **Converting Percents to Fractions** | 1. **Put the percent over a fraction bar.** 2. **100 in the denominator**  * **(You may have to reduce to lowest terms)** * **Ex: 5%= what fraction?** |
| **Converting Fractions to Percents** | 1. **Multiply the fraction by 100/1** 2. **Divide the numerator by the denominator**  * **Ex: 3/5=what percent?** |
| **Multiplying Fractions** | 1. **Multiply numerator by numerator** 2. **Multiply denominator by denominator**  * **Ex: ½ x 2/3=?** * **Ex: 1/3 x 6=?** * **(2/5)2=?** |
| **Dividing Fractions** | 1. **Find the reciprocal of the second fraction** 2. **Multiply the first fraction by the reciprocal of the second**  * **Ex: a/b ÷ c/d =?** |
| **Adding or Subtracting Fractions with the Same Denominator** | 1. **Add the numerators** 2. **Keep the denominators the same**  * **Ex: a/c + b/c =?** |
| **Adding or Subtraction Fractions with Different Denominators** | 1. **Find the least common denominator**  * **Find the least common multiple (the smallest number that is a multiple of both denominators)** * **Ex: 1/3 + 1/4** |
| **Adding Mixed Numbers** | 1. **Follow the same rules for adding/subtracting fractions** 2. **Add the whole numbers separately** 3. **If the result is an improper fraction, you must add the improper fraction to the whole number.**  * **Ex: 3 1/5 + 1 4/5 =?** |
| **Subtracting Mixed Numbers** | 1. **It’s usually best to convert the mixed numbers to improper fractions first** 2. **Then, follow the same rules for adding/subtracting fractions** 3. **Convert to a mixed number, if necessary**  * **Ex: 3 1/5 - 1 4/5 =?** |

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| **Ratio** | * A comparison between two numbers by division * Every fraction is a type of ratio. * A ration can be written with a colon (6:2) or a fraction bar (6/2) |
| The Relationship of Ratios and Fractions | * Every ratio should be reduced to lowest terms. We reduce ratios to lowest terms in the same way as we reduce fractions by canceling out common factors. However, two special rules apply. * Firstly, we *must* have two numbers in our ratio, so we do not convert improper fractions to mixed or whole numbers. For example, in the word problem about the emergency room, the ratio of children to men was 6 : 2. We reduce this ratio to simplest form by canceling out the common factor 2.   =  =  We keep our final answer as 3 : 1. (In other words, we do *not* reduce the improper fraction 3/1 to the whole number 3).   * Ex: Donna processed 68 patient records in a 4-hour shift. Write this quantity as a unit rate (unit rate= ratio reduced to lowest terms). |
| **Analogy** | * A comparison that involves four items (such as people, places, things, or ideas). The first item is related to the second item in the same way that the third item is related to the fourth item * Here’s an example of an analogy that is written in *colon* notation. Colon notation is the way we see analogies written most often.   man : boy :: woman : girl   * Here’s the same analogy written in *fraction* notation:   =  No matter which notation is used, we say the analogy the same way:  *man* is to *boy* as *woman* is to *girl*   * Examples:  1. father : mother :: man : \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ 2. eagle : \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ :: beagle : doghouse 3. water : liquid :: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ : solid 4. \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ : foot :: finger : hand |
| **Proportion** | * Number sentence stating that two ratios are equal. * A proportion is the numerical equivalent of an analogy. * Just as with analogies, proportions can be written in colon notation (4 : 56 :: 1 : 14), fraction notation ( = ), or in words (four is to fifty-six as one is to fourteen). * Just as we do with analogies, we analyze a proportion to see if it’s true or not, and we can solve a proportion to find a missing number. * When the proportion is written in fraction notation, the term *cross multiplication* is used. Cross-multiplying a proportion must reveal equivalency! You’ll often see double-headed arrows pointing to the two means (2 middle numbers) and the two extremes (first and last number) as a reminder of which pairs of numbers to multiply together. Cross multiplication will reveal if a proportion is true or not.   =   * Ex: Is this following proportion true? Why or why not? 18:3 :: 72:12 * Proportions are easier to solve than analogies because the numbers in a proportion are always related to one another by division. * Ex: 39/x = 13/14. Solve for x. * Every set of equivalent fractions (the fractions we make when we raise fractions to higher terms or reduce fractions to lowest terms) is a proportion. * Ex: ½ = x/10. Solve for x. * Whenever we compute percents, we’re really setting up a proportion—the numerator of a fraction is to the denominator as the percent is to 100. (So, 20% = 20/100). * Ex: * Ex: Solve as a proportion: 10% of 60 is what number? * One of the easiest ways to convert from one unit of measurement to another (converting pounds to kilograms, for example) is to use a proportion to solve the problem. * Ex: Solve as a proportion. If 1km=1000m, how many meters is 3.2 km? * When we have a recipe that feeds 8 people and we want to know how much of each ingredient we need in order to make enough to feed the 140 people for a wedding, we use proportions. * Ex: If the recipe that feeds 8 people calls for 3 cups of flour, how much flour is needed for 140 people? * Whenever we have similar figures in geometry (meaning two shapes that are exactly alike except for size), we use proportions to compute any missing dimensions. * Ex: Solve as a proportion. There are two proportional rectangles. The smaller of the two is 3 in. in height by 7 in. wide. The width of the larger rectangle is 21 in., so what is its height? * When we want to know how long it will take us to drive from Boston to Chicago at the rate of 55 miles per hour, we use a proportion. * Ex: If the trip from Boston to Chicago is 1000 miles, how long will it take to drive at 55mph? * Whenever we use a map or a blueprint or any scale drawing, we use proportions to find out how long a distance is or how large an object is in real life. * Ex: Solve as a proportion. Susan is driving to Mount Shasta. On her map, she is a distance of 7 ¾ inches away. The scale of the map is ½ inch = 50 miles. How far must Susan travel to reach her destination? * Proportions are used to solve different kinds of problems in math and should be regarded as one of a math student’s best tools |